...mma can be formulated as follows:
Let S ... discrete set and E a compact (i.e. finite) subset of E. Let $\{\varphi_i(y)\}_1^N$ and $\psi(y)$ be realvalued functions on S with the property that for any given comp. E and for any pos. measure μ with support contained in E, the inequalities

(1) $\int \varphi_i(y)\, d\mu(y) \leq 1 \quad i = 1, \dots N.$

imply

(2) $\int \psi(y)\, d\mu(y) \leq 1.$

Then there exists numbers $\lambda_i = \lambda_i(E) \geq 0$, $\sum_1^N \lambda_i \leq 1$ such that

(3) $\psi(y) \leq \sum \lambda_i \varphi_i(y)$ for $y \in E$.

Let S be a discrete Abelian group and $K(x)$ a realvalued (symm.) function on S such that for $\mu > 0$ with compact support E:

(4) $U^\mu(x) = \int K(x-y)\, d\mu(y) \leq 1$ for $x \in E$

$\Rightarrow$ (5) $U^\mu(x_0) \leq 1$ for each given $x_0 \notin E$.

The basic lemma can be formulated as follows:

Let S be a discrete set and E a compact (i.e. finite) subset of E. Let $\{\varphi_i(y)\}_1^N$ and $\psi(y)$ be realvalued functions on S with the property that for any given comp. E and for any pos. measure μ with support contained in E, the inequalities

$$(1) \qquad \int \varphi_i(y)\, d\mu(y) \leq 1 \qquad i = 1, \dots N.$$

imply

$$(2) \qquad \int \psi(y)\, d\mu(y) \leq 1.$$

Then there exists numbers $\lambda_i = \lambda_i(E) \geq 0$, $\sum_1^N \lambda_i \leq 1$ such that

$$(3) \qquad \psi(y) \leq \sum \lambda_i \varphi_i(y) \quad \text{for } y \in E.$$

Let S be a discrete Abelian group and $K(x)$ a real valued symm. function on S such that for $\mu > 0$ with compact support E:

$$(4) \quad U^\mu(x) = \int K(x-y)\, d\mu(y) \leq 1 \quad \text{for } x \in E$$

$$\Rightarrow (5) \quad U^\mu(x_0) \leq 1 \qquad \text{for each given } x_0 \notin E.$$

By the lemma $\{\lambda_i(E, x_0)\}_1^N$ exists such that

$$K(x_0-y) \le \sum_{y_i \in E}^{N} \lambda_i K(y_i - y) \quad \text{for } y \in E.$$

$$\Rightarrow K(y-x_0) \le \sum_{y_i \in E} \lambda_i K(y-y_i) \quad \text{for } y \in E$$

We conclude that for each finite set F not containing 0 there exists a ~~symm.~~ sym. pos. measure (~~sym.~~) σ with support on F and total mass ≤ 1 such that

$$(6) \quad K(x) \le \int_F K(x-\xi)\, d\sigma(\xi) \quad \text{for } x \in F.$$

The maximum principle for pure potential is therefore ~~equivalent with~~ follows from the property expressed in (6).

The proof of "the basic lemma" can be deduced from "Cartan's lemma": Let $\{a_i\}_0^N$ be vectors in a Euclidean space and have this property: The inequalities

$$(1) \quad (a_i, x) \ge 0 \quad 1 \le i \le N$$

satisfied by x, imply that

$$(2) \quad (a_0, x) \ge 0.$$

Then there exists scalars $\lambda_i \ge 0$ such that

$$(3) \quad a_0 = \sum_1^N \lambda_i a_i$$