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Lemma I

Let $f(t) = \int e^{itx} d\mu(x)$

be pos. def. (real) and $f(0) = 0$. Assume that there

are sequences of numbers $\{t_n\}_1^\infty$ and $\{t_n'\}_1^\infty$

such that $t_n \to \infty$, $\frac{1}{A} < |t_n - t_n'| < A$

$$\lim_{n=\infty} f(t_n) = \lim_{n=\infty} f(t_n').$$

Then $f(t)$ is periodic.

Set $h_n = t_n' - t_n$, $e^{it_n x} d\mu(x) = d\nu_n(x)$

Then $f_n(t) = f(t_n + t) = \int e^{itx} d\nu_n(x)$.

$\int |d\nu_n| \leq 1$; $\int d\nu_n \to 1$. We assume

without loss of generality that ν_n

converges weakly to a pos. measure ν and

that $h_n \to h$, $\frac{1}{A} \leq h \leq A$.

Then $\int e^{ith} d\nu(x) = 1$ and the support of

ν is a subgroup of the real line. The same

set carries μ and the lemma follows.

Corollary I Let $\lambda(n)$ be neg. def. on the integers and $\lambda(0)=0$. Assume that for some fixed $A>1$ there exists arbitrary large numbers p and q $p<q\leq A$ where $\lambda(p)$ λ is arbitrary small. Then λ is periodic.

Apply Lemma 1 to the pos. def. function $e^{-\lambda}$.

Corollary II Let $\Delta_\varepsilon = \{n, \lambda(n)<\varepsilon\}$.

Then the minimal distance L_ε between numbers in Δ_ε tends to ∞ for $\varepsilon \searrow 0$, unless λ is periodic

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Corollary III Let $p(n) = \frac{\lambda_2(n)}{\lambda_1(n)}$, $n \neq 0$, $p(0)>0$. (λ_1, λ_2 not periodic)

The set Δ_ε where ~~either~~ at least one of the inequalities $\frac{\varepsilon}{n^2} > p_n > \frac{n^2}{\varepsilon}$

hold, has the property that $L_\varepsilon \to \infty$ as $\varepsilon \searrow 0$

Lemma II (Conjecture) Let E be a (closed) subset of the circle with the property that an arc of length $2\pi\delta$ does not meet E. If $\eta > 0$ is sufficiently small, then the norms

$$\sum |\hat{\mu}(n)|^2 \frac{\lambda_1(n)}{\lambda_2(n)}$$

and
$$\sum |\hat{\mu}(n)| \cdot \frac{\lambda_1(n) + \eta}{\lambda_2(n) + \eta}$$

are equivalent for all distributions with support on E. (λ_1 and λ_2 not periodic)